# Data and Incentives

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Recent EU regulation draft labels these as "high risk" — not just privacy concerns, but also possibility of the forecasts to **"distort behavior."**

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Our question: How does information extracted from big data impact effort choice and welfare?

# Model

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Total payoff: $\beta \cdot \mathbb{E}[\theta \mid Y] - (1 - \beta) \cdot C(e)$ for some $\beta \in (0, 1)$.

# Equilibrium Effort

The agent's expected payoff given conjectured effort $\hat{e}$ is:

$$U(e;\hat{e}) = -(1-\beta)\cdot\frac{e^2}{2} + \beta\cdot\mathbb{E}^{e}[\mathbb{E}^{\hat{e}}(\theta \mid Y)]$$

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Market's posterior expectation of $\theta$
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Equilibrium effort $e^*$ is uniquely pinned down by the first-order condition:

$$\underbrace{\frac{\beta}{1-\beta}\cdot\left.\frac{\partial\mathbb{E}^{e}[\mathbb{E}^{e^*}(\theta \mid Y)]}{\partial e}\right|_{e=e^*}}_{\text{Marginal value}} = \underbrace{e^*}_{\text{Marginal cost}}$$

# Our Model:

We suppose that $\theta$ and $\varepsilon$ are predictable from primitive covariates:

$$\theta = \theta_1 + \cdots + \theta_J$$
$$\varepsilon = \varepsilon_1 + \cdots + \varepsilon_K$$

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- $\theta_1, \ldots, \theta_J$ are called **attributes**
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Examples:

- Labor market: $\theta$ is productivity, $Y$ is output. "Reliability" is an attribute; "industry shock" is a circumstance.
- College admissions: $\theta$ is ability, $Y$ is GPA. "Attention span" is an attribute; "illness or injury" are circumstances.

# Expansion of Measured Covariates

Some covariates are **measured**, revealing their values for all agents.

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$t = 0$ : Agent and market observe agent's realization $(\theta_{\mathcal{J}}, \varepsilon_{\mathcal{K}})$

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**Big data:** Measured covariates expand from $(\mathcal{J}, \mathcal{K})$ to $(\mathcal{J}', \mathcal{K}')$.

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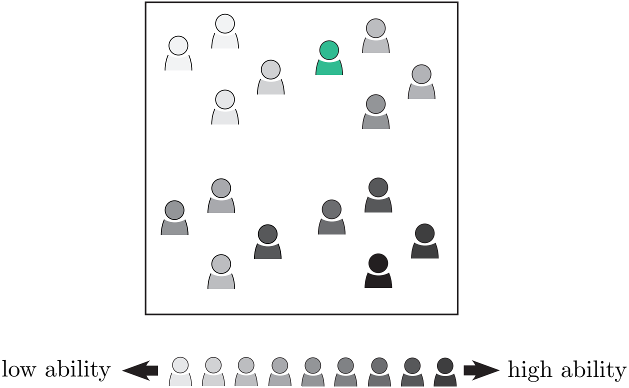
Should the agent know less?

- Agents only need to understand the value of their effort, which may be learned by experience in the market.
- Our results generalize under model uncertainty where the agent is uncertain about the market's understanding of $(\theta, \varepsilon)$.

# What We Ask

How does expansion of measured covariates from $(\mathcal{J}, \mathcal{K})$ to some larger $(\mathcal{J}', \mathcal{K}')$ affect the distribution of effort?



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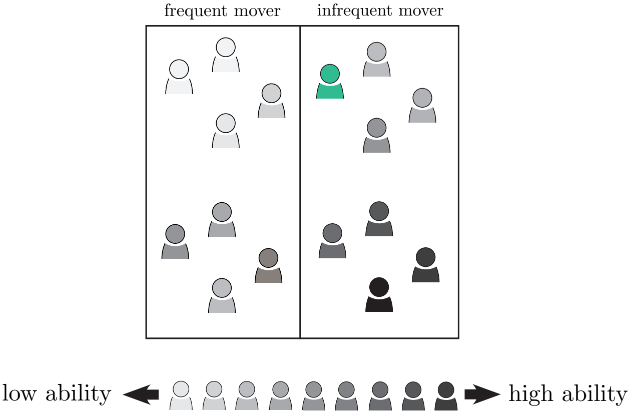

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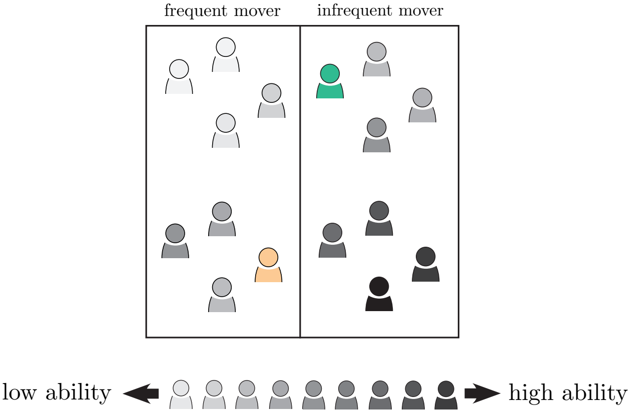

# Related Literature

Growing literature about the economic consequences of big data.

- see e.g. Bergemann, Bonatti, and Smolin (2018); Ichihashi (2019); Bergemann, Bonatti, and Gan (2020); Hidir and Vellodi (2021); Elliot et al. (2021).

Specifically related to us, papers about impact on consumer effort:

- incentives for "gaming" forecasts (Eliaz and Spiegler, 2019; Frankel and Kartik, 2020; Ball, 2020)
- to improve own characteristics (Haghtalab et al., 2020)

These papers treat the data environment as fixed. We vary it.

Methodologically, we build on the career concerns literature.

- Closest paper: Dewatripont et al. (1999). Effort chosen prior to information realizations.

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- Demonstrate a statistical condition which guarantees that disparate impact does not emerge.

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- But the change in effort may differ across agents not only in magnitude but also in sign — we call this **disparate impact**
- Demonstrate a statistical condition which guarantees that disparate impact does not emerge.
- Use these results to determine when measurement of a new covariate improves welfare.

# Illustrative Example

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The two attributes are correlated: $\theta_1 \sim U([0,1])$ and

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Workers with very low residential stability are less reliable on average and also very heterogeneous

- includes individuals who move frequently because of evictions
- Paul Erdős famously had no permanent residence

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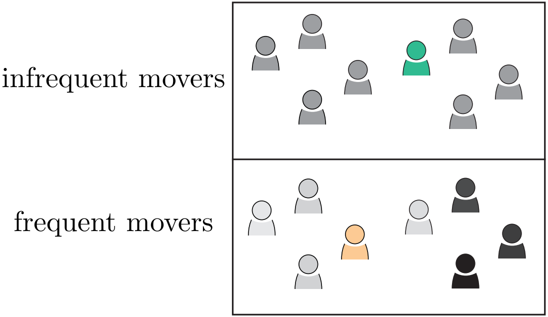
Measuring residential stability leads to **disparate impact**.

# Why Does Disparate Impact Occur?




Measurement of $\theta_1$ **redistributes uncertainty** across agents, with uncertainty about $\theta$ going down for one group and up for another.

- Lower uncertainty about $\theta$ reduces the value to effort
- Higher uncertainty about $\theta$ improves the value to effort

# Main Results

# Data Environments

To simplify notation, suppose in talk that:

$$\theta = \theta_1 + \theta_2$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2$$

Compare the distribution of effort across two data environments:

- No covariates measured vs. $\theta_1$ measured
- No covariates measured vs. $\varepsilon_1$ measured

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- Higher realizations for the measured $\theta_1$ imply higher realizations of the unmeasured $\theta_2$.

**Theorem**

(a) If attribute 1 is Affiliated, then measuring the attribute **reduces** average effort.

(b) If circumstance 1 is Affiliated, then measuring the circumstance **increases** average effort.

# General Intuition

Returns to effort depend on the sensitivity of $\mathbb{E}[\theta \mid Y, \theta_{\mathcal{J}}, \varepsilon_{\mathcal{K}}]$ to $Y$.

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$\longrightarrow$ **Higher** equilibrium effort.

But what measurement of a new covariate implies for **ex post** uncertainty about $\theta$ and $\varepsilon$ is not straightforward.

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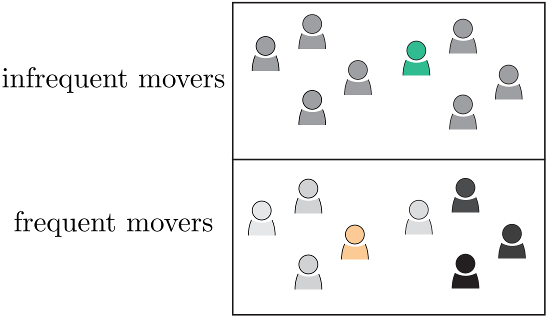
**Indirect** effect (redistribution of uncertainty):

- Conditional uncertainty about $\theta_2$ (given $\theta_1$) may be large, leading uncertainty about $\theta$ to **increase** at some realizations of $\theta_1$
  $\longrightarrow$ higher value of effort at those realizations of $\theta_1$

# Redistribution of Uncertainty



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Previous result says that when a covariate is Affiliated, then the average change in effort is in the same direction as the direct effect.

Next result will further clarify these two forces by shutting down redistribution of uncertainty.

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**Examples**:

- Multivariate normal covariates (with any covariance matrix)
- Additive shifts, e.g. $\theta_2 = X + \theta_1$ for any $X \perp\!\!\!\perp \theta_1$

# Strongly Homoskedastic Covariates

Previous directional change in average effort manifests as a stronger uniform effect:

**Theorem**

(a) If attribute 1 satisfies Strong Homoskedasticity, then measuring the attribute reduces **every agent's** effort.

(b) If circumstance 1 satisfies Strong Homoskedasticity, then measuring the circumstance increases **every agent's** effort.

Moreover the size of change is the same for every agent.

- No disparate impact
- Can interpret as a limiting case where redistribution of uncertainty becomes small.

# Welfare and Regulation

# Which Covariates Should be Permitted?

Suppose measurement of a new covariate becomes available for forecasting

When should the social planner permit the market access to this covariate?

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$$w(\theta, e) \equiv \mathbb{E}^e(Y \mid \theta) - C(e) = \theta + e - \frac{1}{2}e^2.$$

Strictly concave in $e$ and maximized at $e^{FB} = 1$ for every $\theta$.

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$$w(\theta, e) \equiv \mathbb{E}^e(Y \mid \theta) - C(e) = \theta + e - \frac{1}{2}e^2.$$

Strictly concave in $e$ and maximized at $e^{FB} = 1$ for every $\theta$.

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Aggregate welfare is

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where $\mu \equiv \mathbb{E}(\theta)$ denotes the average type in the population.

# Inefficiency of Equilibrium Effort

Given any set of measured covariates $(\mathcal{J}, \mathcal{K})$, the (random) **marginal value of effort** is

$$MV_{\mathcal{J},\mathcal{K}} = \frac{\partial}{\partial e}\mathbb{E}^{e}\left(\mathbb{E}^{e^*}(\theta \mid Y, a_{\mathcal{J}}, c_{\mathcal{K}}) \mid a_{\mathcal{J}}, c_{\mathcal{K}}\right)\Big|_{e=e^*}$$

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Equilibrium effort $e^*_{\mathcal{J},\mathcal{K}} = \beta \cdot MV_{\mathcal{J},\mathcal{K}}$ is FOSD increasing in $\beta$.

- (Recall that agent's total payoff is $(1-\beta)U_1 + \beta U_2$ where $U_1$ and $U_2$ are the payoffs from the two periods.)
- Effort can be inefficiently high or low depending on size of reputation weight $\beta$.

# Regularity

**Definition**

Fix a baseline set of measured covariates $(\mathcal{J}, \mathcal{K})$. Then:

- An attribute $j' \notin \mathcal{J}$ is **regular** if measuring $j'$ decreases average effort.
- A circumstance $k' \notin \mathcal{K}$ is **regular** if measuring $k'$ increases average effort.

All Strongly Homoskedastic and Affiliated covariates are regular (by previous two results).

# Basic Forces

Suppose for simplicity that effort in the baseline and expanded environments deterministic (e.g. if population size $= 1$).

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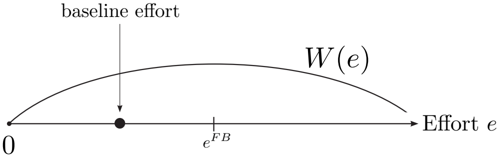

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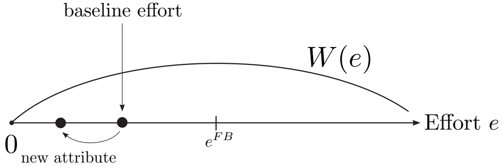


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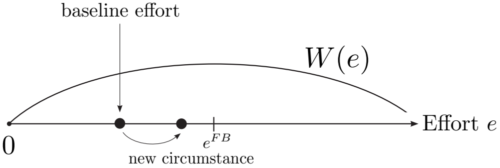


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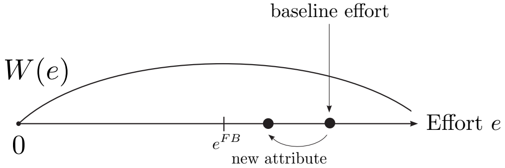


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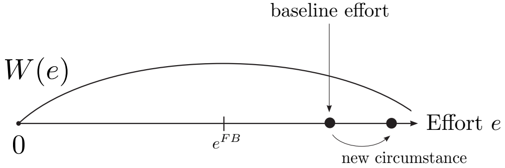


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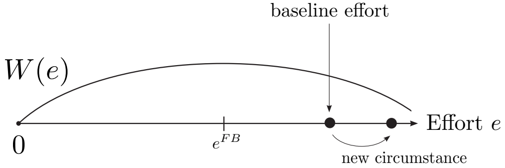


When baseline effort exceeds first-best, a new circumstance always harm welfare.

But more generally, effort is random both in the baseline and after measurement of the new covariate.

# Welfare Impact of New Covariates

**Proposition**

Fix any baseline family of measured covariates $(\mathcal{J}, \mathcal{K})$.

(a) For every regular attribute $j' \notin \mathcal{J}$, there is a $\beta^* \in (0, \infty]$ such that measuring $j'$ improves welfare iff $\beta \geq \beta^*$.

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- Since $\beta_* > 0$, there is always a nondegenerate interval of reputation weights where the circumstance improves welfare.
- But $\beta^* = \infty$ is possible, so the same is not true for attributes.

# Disparate Impact and Welfare Improvement

**Proposition**

Fix a baseline set of measured covariates $(\mathcal{J}, \mathcal{K})$ and a new regular attribute $j' \notin \mathcal{J}$. Then $\beta^* < \infty$ if and only if

$$\mathbb{E}\left(MV^2_{\mathcal{J}\cup\{j'\},\mathcal{K}}\right) < \mathbb{E}\left(MV^2_{\mathcal{J},\mathcal{K}}\right)$$

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## Corollary

In our previous labor market example,

$$\mathbb{E}\left(MV^2_{\{1\},\emptyset}\right) \approx 0.039 > MV^2_{\emptyset,\emptyset} \approx 0.026.$$

Therefore aggregate welfare decreases upon observing attribute 1 for any reputation weight $\beta$.

# Regulation of Covariates: Takeaways

When reputation weights $\beta$ are too low, then **allow circumstances and ban attributes**.

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When reputation weights $\beta$ are too high, then **allow attributes and ban circumstances**.

In either case, disparate impact is harmful—prefer covariates that divide the population up into "similarly well understood" groups.

# Extensions

Model uncertainty:

- Agent is uncertain about observed covariates $(\mathcal{J}, \mathcal{K})$ or subpopulation distributions
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- When should a regulator permit/forbid use of a new covariate in forecasting?

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Our paper tackles two related questions:

- What is the effect of access to new data on reputational incentives for effort?
- When should a regulator permit/forbid use of a new covariate in forecasting?

Future work:

- What data gets collected in a competitive market?
- Endogenous covariates